MATH 54 – MIDTERM 1

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Name: ____

Instructions: This midterm counts for 20% of your grade. You officially have 50 minutes to take this exam (although I will try to give you more time). Good luck, and don't worry, you'll be fine!

Note: Please check the following box if it applies to you:

☐ I am taking a Summer Session A course (May 21 - June 29), and I feel that this has prevented me from showing you my full math potential

1		10
2		10
3		15
4		20
5		15
6		15
7		15
Bonus		3
Total		100

Date: Friday, June 29th, 2012.

1. *(10 points, 2 pts each)*

Label the following statements as **TRUE (T)** or **FALSE (F)**. **Write your answers in the box below!**

NOTE: In this question, you do **NOT** have to justify any answers! Also, don't spend *too* much time on each question!

(a) If the **augmented** matrix of the system $A\mathbf{x} = \mathbf{b}$ has a pivot in the last column, then the system $A\mathbf{x} = \mathbf{b}$ has no solution.

(b) If A and B are invertible 2×2 matrices, then $(AB)^{-1} = A^{-1}B^{-1}$

(c) If A is a 3×3 matrix such that the system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^3$.

(d) The general solution to $A\mathbf{x} = \mathbf{b}$ is of the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_0$, where $\mathbf{x}_p$ is a *particular* solution to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_0$ is the *general* solution to $A\mathbf{x} = \mathbf{0}$.

(e) If P and D are $n \times n$ matrices, then $det(PDP^{-1}) = det(D)$

(a)	
(b)	
(c)	
(d)	
(e)	

2. *(10 points, 5 points each)* Label the following statements as **TRUE** or **FALSE**. In this question, you **HAVE** to justify your answer!!!

   This means:

   - If the answer is **TRUE**, you have to explain **WHY** it is true (possibly by citing a theorem)
   - If the answer is **FALSE**, you have to give a specific **COUNTEREXAMPLE**. You also have to explain why the counterexample is in fact a counterexample to the statement!

   (a) If A and B are any 2×2 matrices, then $AB = BA$

   (b) The matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$ is not invertible.

3. *(15 points)* Solve the following system of equations (or say it has no solutions):

$$\begin{cases} 2x + 2y + z = 2 \\ 3x + 4y + 2z = 3 \\ x + 2y - z = -3 \end{cases}$$

4. *(20 points)* Solve the following system $A\mathbf{x} = \mathbf{b}$, where:

$$A = \begin{bmatrix} 1 & 1 & 1 & -3 \\ 2 & 3 & 1 & -6 \\ -1 & 2 & -4 & 3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 8 \\ 3 \end{bmatrix}$$

Write your answer in (parametric) vector form

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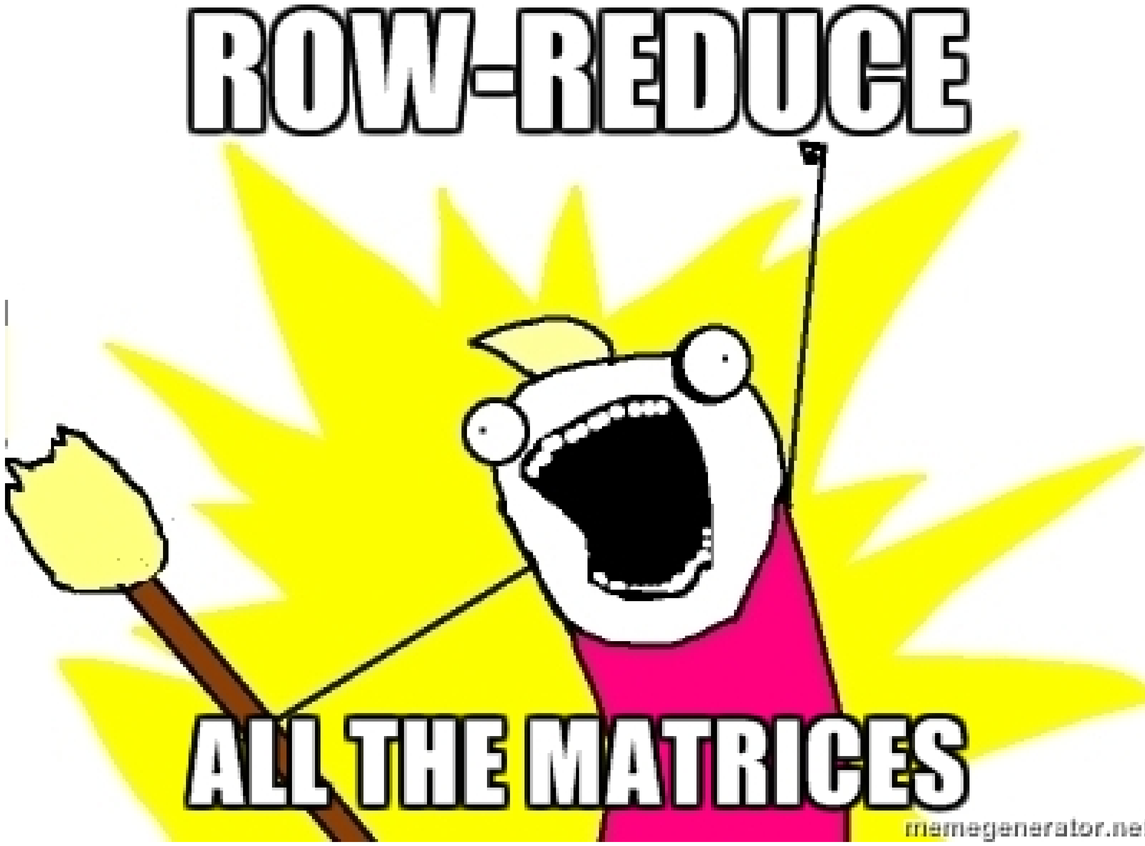

5. *(15 points, 5 points each)*

(a) Calculate AB, or say that AB is undefined.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

(b) Calculate AB, or say that AB is undefined.

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 3 & 0 \end{bmatrix}$$

(c) Calculate A^2, where:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

6. *(15 points)* Find A^{-1} (or say 'A is not invertible') where:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

7. *(15 points)* Find $det(A)$, where:

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 & 1 \\ 2 & 0 & 4 & 0 & 5 \\ 1 & 2 & 5 & -2 & 0 \\ 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

Bonus *(3 points)* Find $det(A)$, where:

$$A = \begin{bmatrix} 1 & x & x^2 & x^3 \\ 1 & y & y^2 & y^3 \\ 1 & z & z^2 & z^3 \\ 1 & t & t^2 & t^3 \end{bmatrix}$$

where x, y, z, t are distinct real numbers. This is called a **Vandermonde** matrix!

Hint: Calculating this directly is going to drive you nuts! Can you think about another way of calculating the determinant?

Note: You might find the formula $p^3 - q^3 = (p-q)(p^2+pq+q^2)$ (where p and q are real numbers) useful!

(Scratch work)

Any comments about this exam? (too long? too hard?)